Data metrics in modern digital marketing

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Abstract. The article defines the stages of digital marketing and highlights modern metrics for choosing data analysis tools. Currently, mobile Internet accounts for 80% of the total Internet traffic in the Russian Federation. The research emphasises the components of effective media planning, including return on investments, target audience, increasing brand awareness, and costs optimising. According to the statistics, there are different approaches to increasing sales of new trademarks (brands) via the mobile Internet. The research considers the issues of calculating the market share of a trading company, advertising voice among the voices of other brands, and calculating the advertising budget for promoted similar budgets of other companies. The paper proves the validity of the formula expressing the equilibrium in a market with several players. The social networks, messengers, digital television, and neural networks replace the traditional advertising channels. It provides the reduction of brand forgetting time. Moreover, the traditional advertising impact on the target audience is losing its effectiveness. The considered mathematical model of the advertising response redistribution shows the inverse dependence of the share of the advertising voice depending on the number of the promoted product in the company's product line. Other data metrics allow ones to increase sales profits in addition to the main advertising strategy of the enterprise through geographic information systems (coverage radius), customer comments, and reviews on marketplaces and digital cinemas. The use of big data technologies transforms the methodology of effective advertising. It applies the econometric laws providing a "target funnel" for product promotion and Nevertheless, it requires large budgets to maintain the promoted brands. Therefore, the marketing services of enterprises investigate market segmentation and assess the advertising budgets of competitors. It maximises the effectiveness of an advertising campaign following the entry of competing companies into the same market. Hence, the share of the regional or global market of the promoted product is proportional to the advertising budget relative to the total budget of the entire market. The funding for the promotion of a new brand for well-known purposes is an analytical dependence of Peckham. The individual share of the advertising vote (out of 100% of votes for new brands) analytically depends on the company's historical market share and the number of the new brand for the reporting period (1, 2, 3, etc. years). The individual share of the advertising voice is a dimensionless random variable depending on the relative frequency of the brand over the previous period. The integral calculus in advertising forecasting provides marketers with a powerful tool for analysing data and making informed decisions. Such models include the Bass and the Adstock models. Probability metrics are important tools for assessment of advertising campaigns effectiveness and making strategic decisions. They allow ones to consider the uncertainties and randomness characteristic of consumer behavior. Moreover, the game theory is a powerful tool for analysing and developing advertising strategies. It allows companies to consider the behavior of competitors, respond to changes in the market environment, and make informed decisions. The use of game theory helps to minimise risks and maximise the benefits of advertising campaigns, ensuring sustainable business development.

Keywords: digital marketing; media planning; online advertising; advertising optimisation; data metrics; mathematical modelling

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Introduction

The purpose of the study is to define and initialise the metrics of advertising campaigns in terms of the digital marketing perspective. The main objectives are to examine various data metrics in target audience allocation, study Peckham's formula – a mathematical model for optimising advertising share, propose trends of studying of advertised product effectiveness through big data technologies.

Media planning is the process of developing a strategy for placing advertising messages in various media channels to achieve the company's marketing goals. The main goal of media planning is to maximise the effectiveness of the advertising budget by reaching the target audience properly.

The main stages of media planning:

1 Market and target audience analysis. This stage involves market research, analysing the requirements and preferences of the target audience. It is important to understand the potential buyers of a product or

service, their interests, habits, etc.

2 Setting goals. The goals may involve increasing brand awareness, boosting sales, attracting new customers or strengthening the loyalty of existing ones. They should be specific, measurable, achievable, relevant, and time-bound (SMART).

3 The choice of communication channels. Depending on the target audience and objectives, the most appropriate channels for advertising distribution are selected. These can include television, radio, press, outdoor advertising, internet (contextual advertising, social networks, banner advertising), mobile applications, and other platforms.

4 Budget development determines the optimal allocation of funds between different channels. It considers the cost of advertising, possible additional costs, such as the development of creative materials, analytics, and performance monitoring

5 Development a media plan: a detailed plan of advertising placement is developed with specific dates, times, and places of advertising messages. The media plan includes publication schedule, number of impressions, frequency of audience contacts, etc.

6 Implementing the plan: after the media plan approval, the implementation stage begins. Advertising materials are placed in accordance with the developed schedule, their effectiveness is monitored and, if necessary, adjustments are made.

7 Monitoring and assessment of the results. An important part of media planning is to constantly monitor the implementation of the plan and analyse its effectiveness. Various metrics such as a media outreach, click-through rate (CTR), conversion rate, etc. are used to assess the success of the campaign.

The advantages of competent media planning:

1 Maximising return on investment (ROI): efficient use of the advertising budget allows ones to achieve greater results at lower costs.

2 Target audience: the right choice of advertising channels and formats helps to deliver the message to the people really interested in it.

3 Increasing brand awareness: regular presence in the media space helps to strengthen the company's image and increase its popularity among consumers.

4 Cost optimisation: competent allocation of resources helps to avoid unnecessary expenditure.

Therefore, media planning plays a key role in the success of any advertising campaign. It requires deep analysis, strategic thinking, and constant monitoring of results to ensure maximum ROI.

However, the numerical assessment of media planning indicators have been relevant in Russia since the 2000s [11]. To optimise spending on advertising campaigns, retailers used both their own financial capabilities and the budgets of regional representatives of the Western brands. At that time, the advertising media channels were television, outdoor advertising, news papers, and radio. The Internet in Russia started e-mailing and advertising in social networks. The advertising market has been transformed greatly. Indeed, Aliexpress initiated the sales into the Internet. Nowadays, the monopoly services: Ozon, Yandex, Sber, etc. have their own ecosystems. Moreover, the monopolists share the digital market. It was boosted by the COVID-19 pandemic. However, the digitalisation of marketing communications is affected by the contextual advertising, search engine advertising, smartphone apps, computer games, etc. By the end of 2020, Russians have already increased the total turnover through mobile apps to $1.33 bn USD (25% of total purchases). People are in overall communication. Digital wellbeing today correlated with the challenge of protecting a person's personal space. In 2021, the share of mobile Internet in Russia reached 81% (against 19% for other devices)[1]. Therefore, the promotion of goods and services through smartphone applications is becoming more and more relevant.

However, digital marketing and sustainable development emerge new trends and innovations as companies increasingly realise the importance of environmental and social responsibility towards citizens. Therefore, many studies have analysed trends indicating significant changes in consumer preferences towards environmentally friendly products and socially responsible behaviour. The paper provides examples of several international campaigns illustrating successful efforts to promote sustainable development. It discusses the

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[1] *The mobile economy of Russia. Source: https://www.tadviser.ru/index.php (accessed on 19.10.2024)*

role of digital technologies in achieving sustainability goals and emphasises the necessity of integrating sustainability issues into corporate digital strategies [9]. One of the trends in digital marketing is the carbon footprint indicator to save biological resources. However, this approach is controversial but effective one: Aeroflot and Avito implement those indicators on their sales page.

Some researchers use psychological approaches in media planning. Indeed, people eventually forget a picture or text they have seen. To strengthen the stimulus signal of a media carrier there is a practice to repeat it again at the moment of forgetting [8]. The method based on forgetting coefficients as the target function of minimising advertising costs. The approach to mathematical modelling.

The scalar product of vectors method is the basic one in terms of feedback metric for the film and television industry. For instance, the several users rate a film from 1 to 10. Therefore, we introduce a point on the Cartesian plane with abscissa – the number of positive ratings for the film 1. On the ordinate axis we will mark the number of positive ratings for film 2. If we construct two radius vectors with ends at the constructed points, the measure of polarity (quality) of the films is the scalar product between these radius vectors, or by the cosine between them. The closer the cosine is to zero, the higher the polarity, and the closer the cosine is to one, the lower the polarity. We can use the similar metric for the entertainment sector, feedback assessment, in service and tourism, banking and medical activities, education, etc.

Another approach in defining the metric is to apply geographic coordinates (GIS-metric). The distance between points on the earth's surface is approximately the same as the Euclidean distance between currents on a plane. Among the tasks of performance assessment with this metric is the distance from the property to the metro station in large cities, to railway and bus stations, major motorway routes. In this case, the value of the object directly depends on the distance. In the topic of our study another form of distance is more important – the radius of a circle with the centre in the studied point of capital growth to nearby objects. Coverage of residential buildings from Yandex.Market or Ozon parcel delivery point. E-businesses of such companies use geodata to promote their goods and delivery services within a given radius of service coverage. The Russian Presidential Academy of National Economy, St. Peterburg, Russia conducted geometrics research for Gazprom during the construction of the Lakhta Centre. The geo-location of the centre within a radius of 10 km was used for the development of transport infrastructure (metro stations, railway overground stations, airport, motorways), social infrastructure, and housing for the employees. They include the construction of the Ust-Luga seaport, the satellite town of Yuzhny, the relocation of industrial enterprises from the outskirts of the city to the Leningrad region, etc.

Main part

The fundamental research on the dissemination of advertising as information on a product or service interested Russian scientists in the 90s of the last centuries. Media planning has two important objectives: optimisation of advertising placements in terms of reach (the share of the target audience expressed as a percentage) and in terms of placement cost (monetary value of the contract per unit of time) [18, 6]. We understand optimisation as the efficiency of the advertising campaign (to get the maximum increase in sales during the advertising campaign period for a minimum of added funds).

At the first stage of planning it is difficult to compare such a process with optimisation in a mathematical point of view, because many components of the advertising industry are stochastic (in essence, random variables or random processes).

However, some studies consider the mathematical problem of assessing the effectiveness of advertising campaigns. Based on the analysis of world experience and empirical data, we designed the mathematical models to predict the level of awareness of a brand. It substantiates the use of heuristic search methods as the most appropriate tool for processing sociological survey data, in terms of the principles of complex systems behaviour [12]. Moreover, it is an effective tool for providing the econometric analysis of advertising response data (real purchases). This tool is effective only with a large number of trials (more than 100 sales). The errors e of any linear model are $y=a_0+a_1x+e$ *distributed* according to a normal law, which is asymptotic for any other distribution. There are usually a large number of players in the market, so the distribution of advertising does

not spread linearly. Rather, it is a non-linear process. However, there is a different approach.

Popular advertising channels since 2015 include browsers, instant messengers, games, smartphone apps, and social networks. The Peckham method is used to numerically assess the advertising budget of a new brand with the share of the advertising voice S(k) as the product of the coefficient α (taken at the level from 1.5 to 2, a dimensionless value) by the intended share of sales X(k). It is proposed as a limited function with parameter n – the ordinal number of the new brand in the product line presented in formula (1) [18, p.51]:

$$S\left(k\right) = \frac{\alpha \cdot k^{n-1} \cdot (1-k)}{1 - k^n} \tag{1}$$

The advertising budget according to formula (1) depends on the constant k – the market share of the previous product of the specified company. Number n is the number of the current brand from all the brands of this seller. By the type of function, it is a power function. This function is resistant to variable changes and is easily differentiated or integrated in composing more complex models based on the theory of differential equations.
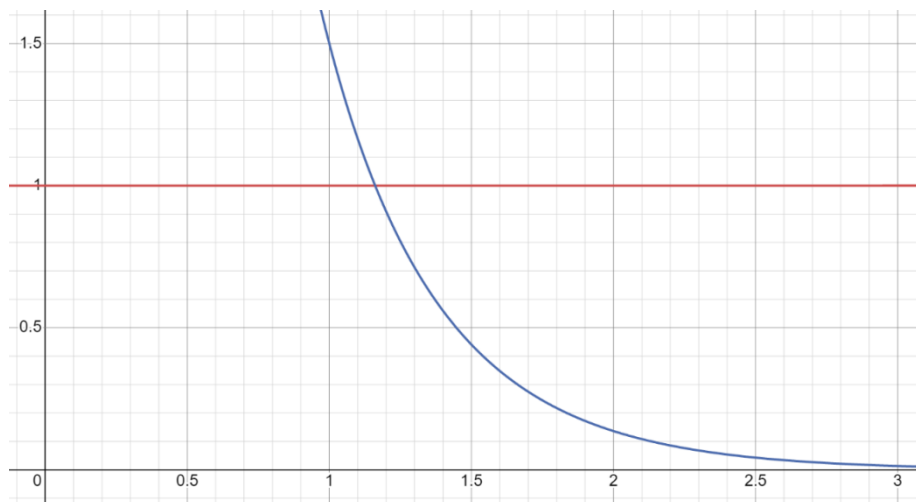

Figure 1. Advertising voice share S(n,k). k=0.1

The considered mathematical model shows the inverse dependence of the share of the advertising voice depending on the number of the promoted product in the company's product line. One of the main goals of marketing is to optimise (minimise) the advertising budget through reaching the largest possible target audience.

Media planning optimization is divided into two groups:

Group 1. Maximising the share of advertising voice S or effective reach G or average contact frequency f or projected profit P;

Group 2. Minimising the advertising budget of R at maximising one of the indicators specified in paragraph 1.

We calculate the S values for practical values from the source [1] for advertising several services promoted through mobile applications. The values of the share S(k) of the advertising voice of a company producing "black" household appliances with a market share k=0.1 for products already sold with numbers n of the product line being sold (it takes natural values from 1 to 10) have the graph shown in Figure 1. For more significant brands with a high k market share (from 0.5 to 0.9), the result will be different (Figure 2).

According to Figure 2, the share of advertising voice is higher the younger the company is as the market prefers new brands; for the monopolists (with a market share of 0.5), the share of advertising voice is achieved at a lower cost than for newcomers in this market segment. The schedule allows ones to minimise the cost of an advertising campaign for major players. Therefore, we examine the extremum function on the interval [0.5;0.9]. The extremum points are the points shown in Table 1. The table shows the values of the function S(n) for these different n.

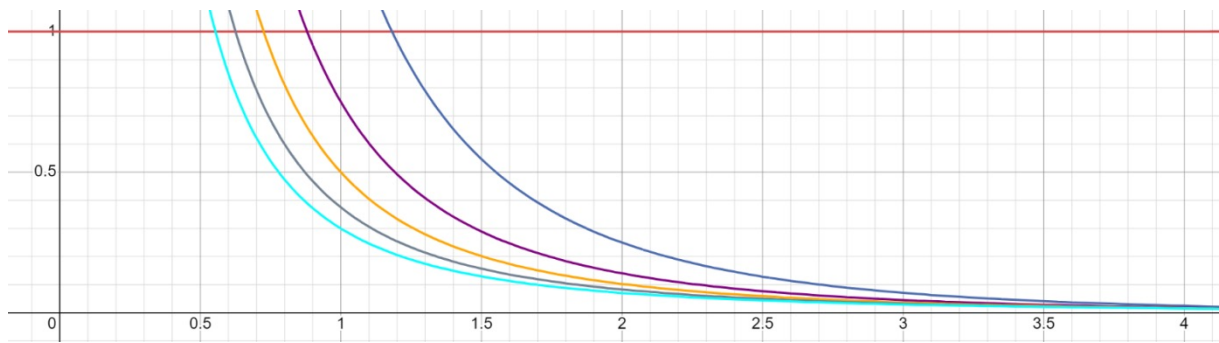**Figure 2.** The share of the advertising voice S(k, n) for k>0.5, n is a natural number, the Peckham formula
Source: Author

Table 1 – Maximum values of the function S(n, k) – the share of advertising voice according to the Peckham formula

n	1	2	3
k	0.50	0.61	0.69
S(n,k)	1.5	0.59	0.33

Source: Author

According to the calculated values of the function for a new brand, the voice share will be much higher (within 1.5) than for newly released product lines (less than 0.2). Moreover, it requires large budgets to maintain the promoted brands. In this case, the marketing services of enterprises investigate market segmentation and assess the advertising budgets of competitors. It maximises the effectiveness of an advertising campaign following the entry of competing companies into the same market.

Indeed, the owner of a new brand can maintain its market share, as long as the share of advertising voice should be equal to the share of advertising budget in the volume of competitors' advertising budgets. We obtain equality (2):

$$V = \frac{X}{1 - X} \sum_{j=1}^{m} V_j \tag{2}$$

According to [5], the advertising budgets for the promotion of smartphones of four well-known brands on the mobile network were obtained: V1=$ 30 mln USD, V2=$ 17 mln USD, V3=$ 10 mln USD, V4=$ 12 mln USD. However, over the past 3 years, for the first and second companies, this is product No. 3 (n=3); for the third company this is brand No. 2 (n=2); the fourth company is entering the market for the first time (n=1). Using formula (1), we calculate the value of the share of sales for company No. 3: X3=X(3;0.61)=0.1877. The economic issue of this coefficient is as follows: when promoting a new product, the company will gain a market share of similar products in the region of 18.77%. To find out the reasonable size of the advertising budget for the promoted project No. 3, we use formula (2) $V = \frac{X^3}{1 - X^3}(V_1 + V_2 + V_4) = 13{,}63$. However, there is

an error in the equation. In this case, the savings will amount to about $ 3.63 mln USD. The relative margin of error will be 5%. This is confirmed by similar calculations in [5, 7].

Formulation of the inverse problem: based on the data of the advertising budgets of the selected company and the advertising budgets of three competing companies, we find the share of sales (increase) in relative terms. For these purposes, we consider formula (2) as an equation with respect to the variable X (share of sales). The advertising budgets are shown in Table 2:

Table 2 – Parameters of the equation of balance of advertising budgets to identify the share of sales of the target enterprise

№	Advertising budget, $ mln USD	Share of sales, X
1	30	0.43478

№	Advertising budget, $ mln USD	Share of sales, X
2	17	0.24638
3	10	0.14493
4	12	0.17391
amount	69	1

Source: Author

Equation (2) provides an analytical solution as a rational equation with respect to the variable X: the share of sales out of the total number of smartphones sold will be proportional to the advertising budget for promoting this model. This is symbolically expressed by formula (3):

$$X = \frac{V_j}{\sum_{i=1}^{m} V_i} \tag{3}$$

The sum of all the shares calculated by formula (3) will be equal to one (100%). This proves the validity of formulas (1) and (2).

Further research in this area uses risk theory in media planning [17]. Apparently, the risk here is considered stochastic; the amount of profit from an advertising campaign is a random variable. However, the complexity of calculations increases to the extent; it is easier to order the development of a media plan from a specialised company, for example, Yandex. Direct. In this case, the company independently predicts the outcome of the advertising campaign using artificial intelligence services for a fixed amount.

We have reviewed functional modelling in advertising. Integral calculus is a more complex tool. Integral calculus is a powerful mathematical tool applied in various fields of science and technology, including economics and marketing. In the context of advertising forecasting, this method is used to assess changes in indicators such as sales volume, demand level, audience response to advertising campaigns, etc.

The main aspects of using integral calculus in advertising forecasting are as follows:

1. Assessment of the accumulated effect of advertising. Advertising has a cumulative effect – its impact accumulates timely. The integral allows ones to calculate the total reaction of the audience to an advertisement over a certain period of time. For instance, if the function f(t) describes the daily change in the level of interest in a product under the influence of advertising, then the integral of this function over time will show the total interest over a period of time [16].

2. Forecasting sales volume. The volume of sales often depends on the intensity of the advertising campaign. Using data on previous campaigns and related functions, ones can build a model forecasting future sales based on planned advertising activities. The integration of these functions will make it possible to obtain the projected sales volume for a certain period [2].

3. Optimisation of advertising budgets. Integral methods help to determine the optimal distribution of advertising budgets between different channels and advertising formats. By analysing the effectiveness of each advertising activity through integrals, it is possible to maximise the profits through a balance between costs and results [3].

4. Product lifecycle modelling. The product lifecycle has several stages: introduction, growth, maturity, and decline. Integral calculus allows ones to model these stages and forecasr sales behavior at each stage. For instance, to calculate the integral of a function describing the demand for a product in order to estimate the total revenue over the entire lifecycle [10].

5. Assessment of seasonal fluctuations. Many goods and services are subject to seasonal fluctuations in demand. Therefore, we can use the harmonic functions and Fourier series. The integrated approach allows ones to consider these factors and adjust forecasts to assess real market conditions. For instance, to integrate a function in terms of the seasonal fluctuations for more accurate results.

The Bass and Adstock models can serve as examples of the use of integral calculus in advertising. The Bass model is a differential equation describing the spread of innovation. It uses integrals to calculate the cumulative number of new product buyers. The ordinary differential equation of Bass is represented in

formula (4).

$$\frac{dF}{dt} = p(m - F(t)) + q\frac{F(t)(m - F(t))}{m} \tag{4}$$

where F(t) is the cumulative number of accepted innovations on time t, m is the total number of potential users (market capacity), p is the innovation coefficient (coefficient of external influence), and q is the imitation coefficient (coefficient of internal influence).

The innovation coefficient p reflects the proportion of consumers accepting a new products independently of others due to external factors such as advertising, promotion, and PR. These people tend to try new things independently. The imitation coefficient q shows the proportion of consumers accepting a new product under the influence of other people already using this product. The more people accept the product, the stronger is the social pressure and desire to join the majority. The Bass model helps companies to forecast the sales growth rate of a new product based on initial conditions and parameters p, q, and m. However, data collection is usual for model use. It is necessary to collect historical data on sales of similar products or conduct preliminary market research to estimate the values of the parameters p, q, and m; parameter selection is also in use. It includes data collecting, the coefficients p and q, and the market capacity m. This can be done manually or using statistical methods such as the least squares method. When the parameters are determined, we can use the model to forecast the future sales. Therefore, we solve a differential equation with respect to F(t) and determine the rate of adoption of a novelty S(t) at each moment in time. Analysis of the results: the forecasts are compared with real data to verify the accuracy of the model. In case of insufficient results, we can repeat the steps of parameter selection and forecasting. The Bass model remains one of the most important tools in the arsenal of marketers and product managers. It allows ones to accurately forecast the sales dynamics of new products and make informed decisions about marketing and promotion strategies. Despite its limitations, this model continues to be relevant and useful for many companies attempting to successfully introduce their innovative offers to the market. However, the model is difficult to calculate and requires the specialised software [3].

The Adstock model's ad response feature suggests exposure to an advert persists for some time after the advert has been completed. The integration of this function provides an assessment of the long-term effect of an advertising campaign. The ad response function, known as the Adstock model, is a mathematical approach to assessing the impact of advertising on consumer behavior. This model is based on the assumption that the effect of advertising persists and gradually weakens over time. The Adstock model allows ones to consider the residual effects of past advertising efforts and forecast future consumer behavior. The main components of the Adstock model are as follows: residual memory; the rate of decay (forgetting of an advertising message), the level of advertising impact determines how strongly current advertising efforts affect consumers (expressed as frequency of impressions, audience reach, or other indicators), and the half $T_{(1/2)}$-life – the time it takes for the initial effect of the advert to be halved. The model is given by the recurrence equation:

$$A_t = E_t + \lambda A_{t-1} \tag{5}$$

where is the A_t- current level of exposure to advertising, E_t- the exposure level, $A\_{(t-1)}$ - the previous level of exposure to advertising. Therefore, we can calculate the current value of the advertising impact. The advantage of this model is the ease of calculation even in a simple spreadsheet [14].

Probabilistic metrics in advertising are diverse and used in statistical probability, game theory, and Markov processes. Statistical relationships are obviously the most popular in the marketing environment as they are easily understood by any professional. Customisations and definitions in digital marketing and advertising are based on these relationships. Probabilistic metrics play an important role in advertising, assisting in evaluating the effectiveness of advertising campaigns and making informed decisions. Generally, probabilistic approaches allow ones to consider the uncertainty and randomness inherent in consumer behaviour and markets. Click-Through Rate (CTR) is one of the main metrics in digital advertising showing the probability of a user clicking on an ad; it is calculated as the ratio of clicks to user impressions. A high CTR

level indicates the attractiveness and interest in the message. Another metric, the CR conversion probability, assesses the probability of performing the targeted action (conversion) by the user after interacting with an ad. CR is calculated as the ratio of the number of conversions to the number of clicks or to the number of impressions. Return Probability RP assesses the probability of a user returning to a site or app after a first visit; it is the ratio of the number of returns to the total number of new visitors to the site or app. The probability of repeat purchase RPP is the number of repeat customers to the total number of all customers. A high probability indicates customer loyalty and satisfaction with a purchase or service. Indeed, over the last 2-3 years, more and more companies are using reward cards, subscriptions, and apps to retain loyal customers. There are also assessments of negative phenomena in advertising: churn probability, abandonment probability, and other metrics [19].

Game theory is a model popular in the domestic school of marketing. This branch of optimisation methods came into science back in the last century, but has only become widespread in the modern economy in the last 10 years. Companies use game theory to analyse the behaviour of their competitors and develop strategies to take a leading position in the market. For instance, two companies may choose different strategies to promote their product: one may focus primarily on television and the other on digital platforms. Game theory can help determine the strategy most beneficial depending on competitor's actions. Companies can use game theory to determine the optimal price for their products. For instance, one company cuts its price, another company is required to decide whether it should also cut its price or keep it the same. The payment matrix will help ones to assess the consequences of each option implementation. The choice of promotion channels (television, radio, Internet, social networks, etc.) can also be analysed in terms of game theory. Companies can consider the most effective channel combinations in terms of competitors and the preferences of the target audience. Game theory can help companies decide on partnerships or alliances with other companies. The payment matrix will show the option of cooperation resulting in the best results for all participants [4, 20, 15].

Moreover, there are probabilistic chains. The Markov chain is used to describe transitions between states (for example, a customer goes from a state of "not aware" to a state of "aware"). By integrating the transition probabilities, it is possible to predict the dynamics of audience behaviour.

Conclusion

According to research results, the following conclusions can be drawn:

1 The share of the regional or global market of the promoted product is proportional to the advertising budget relative to the total budget of the entire market – formula (3); the market share is not a random variable; the amount of financing for the promotion of a new brand for well-known purposes: the share of the advertising voice and the advertising budgets of competitors is an analytical dependence of Peckham – formula (2) with a relative margin of error in the region of 5%; the amount of financing is not a random variable; the individual share of the advertising vote (out of 100% of votes for new brands) analytically depends on the historical market share of the company and the number of the new brand for the reporting period (1, 2, 3, etc. years);

2 The individual share of the advertising voice is a dimensionless random variable depending on the relative frequency of the brand over the previous period; it is linked to sales statistics [13].

3 The integral calculus in advertising forecasting provides marketers with a powerful tool for analysing data and making informed decisions. This method allows ones to consider many factors affecting the effectiveness of advertising campaigns and make more accurate forecasts. It causes cost optimisation and increased return on investment in advertising. Such models include the Bass and the Adstock models.

4 Probability metrics are important tools for assessment of the effectiveness of advertising campaigns and making strategic decisions. They allow ones to consider the uncertainties and randomness characteristic of consumer behavior, more accurately predict the results, and optimise advertising budgets.

5 Game theory is a powerful tool for analysing and developing advertising strategies. It allows companies to consider the behavior of competitors, respond to changes in the market environment, and make informed

decisions. The use of game theory helps to minimise risks and maximise the benefits of advertising campaigns, ensuring sustainable business development.

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